Advanced Placement Statistics
Chapter 10 Review Problems

Name: Mr. Morton
Date: ________ Period ________

1) Seventy-seven students at the University of Virginia were asked to keep a diary of conversations with their mothers, recording any lies that they told during these conversations (San Luis Obispo Telegram-Tribune, August 16, 1995). It was reported that the mean number of lies per conversation was .5. Suppose the standard deviation was .4.

PANIC

a) Assuming that the sample of 77 students was a simple random sample, construct a 95% confidence interval for the mean number of lies per conversation for this population.

P	Parameter of interest, true μ for number of lies per conversation.
A	SRS ✓, $n=77 \geq 30$ Approx. Normal (CLT) ✓, Population $\geq 10(77)$ ✓
N	95% confidence interval (z-interval).
I	$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} = 0.5 \pm 1.96\,(.4/\sqrt{77}) \rightarrow (0.4107, 0.5893)$.
C	We are 95% confident that true μ lies between 0.4107 and 0.5893.

b) In the interval in part (a) does not include zero. Does this imply that all students lie to their mothers? Explain.

No, just the majority of students have lied to their mothers.

c) What sample size would be needed to decrease the margin of error to .05?

$z^* \frac{\sigma}{\sqrt{n}} \leq m$ | $1.960 \frac{0.4}{\sqrt{n}} \leq 0.05$ | $n \approx 246$ students

2) Samples of two different types of automobiles were selected, and the actual speed for each car was determined when the speedometer registered 50 mph. The resulting 95% confidence intervals for true average speed were (51.3, 52.7) and (49.4, 50,6). Which confidence interval is based on the larger sample size? Explain your reasoning.

X 52 ± 0.7, 50 ± 0.6 | larger sample size, smaller margin of error.

3) You read an article that describes a study of the voting patterns of various groups in society based on a large sample survey. The article says, "Persons who identified themselves as evangelicals were significantly (P < 0.1) more likely to favor Republican presidential candidates that were other white Protestants." Explain to someone who knows no statistics what "significantly (P, ~~.1~~)" means.

(P, 0.10)

This means that the evidence againt H_0 is so strong that it would happen no more than 10% of the time.
(1 time in 10 samples)

Phantoms! $\alpha = 0.05$

4) During the National Football League (NFL) season, Las Vegas oddsmakers establish a point spread on each game for betting purposes. For example, the Baltimore Ravens were established as a 3-point favorite over the New York Giants in the 2001 Super Bowl. The final scores of NFL games were compared against final point spreads established by oddsmakers in *Chance* (Fall 1998). The difference between the game outcome and point-spread (called a point-spread error) was calculated for 240 NFL games. The mean and standard deviation of the point spread errors are $\bar{x} = -1.6$ and $s = 13.3$. Use this information to test the hypothesis that the true mean point spread error for all NFL games is 0. Conduct a test at the significance level and interpret the result. You may assume that $\sigma = s$.

P We want to test a claim for true μ for point spread error.

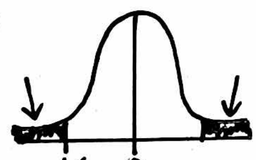
H $H_0: \mu_0 = 0$, $H_a: \mu_a \neq 0$

A SRS? (PWC), $240 \geq 30$ Approx. Normal CLT

N One Sample Z-Test | $N > 10(240)$

T $\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{-1.6 - 0}{13.3/\sqrt{240}} = -1.8637$

O



-1.6 0

P-value = 0.03118

$2(0.03118) = 0.0624$

M/S Since $0.0624 > 0.05$, we fail to reject H_0. Therefore, no evidence against $\mu = 0$.

5) The EPA sets a limit of 5 parts per million (ppm) on PCB (a dangerous substance) in water. A major manufacturing firm producing PCB for electrical insulation discharges small amounts from the plant. The company management, attempting to control the PCB in its discharge, has given instructions to halt production if the mean amount of PCB in the effluent exceeds 3 ppm. A random sample of 50 water specimens produced the following statistics: $\alpha = 0.05$
$\bar{x} = 3.1$ ppm and $s = .5$ ppm (use this for σ)

a) Do these statistics provide sufficient evidence to halt the production process? NO!

$H_0: \mu = 3$ | $Z = \frac{3.1 - 3}{.5/\sqrt{50}} = 1.4142$ | P-value = 0.07865 | Since $0.07865 > 0.05$, we fail to reject at the 5% SL.
$H_a: \mu > 3$

b) Define Type I and Type II errors in the context of the problem.

Type I: When mean amount is 3 ppm, but we conclude there is more.

Type II: When mean amount is greater than 3ppm, but we conclude exactly 3ppm.

c) If you were a plant manager, would you want to use a large or small value for α for the test in part (a)? Explain.

Small α to make sure you do not have a Type I error.

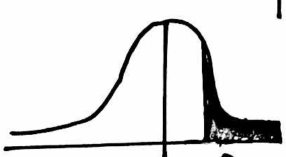
d) Calculate the probability of Type II error assuming that the true mean is 3.1 ppm. $\alpha = 0.05$

$1.645 = \frac{\bar{x} - 3}{0.5/\sqrt{50}}$

$\bar{x} = 3.1163$

$Z = \frac{3.1163 - 3.1}{0.5/\sqrt{50}} = 0.231$

Calc: 0.4088



3.1 $\beta = 0.4088$

e) What is the power of the test to detect the effluent exceeding 3.0 ppm when the true mean is 3.1ppm?

$1 - \beta = 0.5912$

Directions: Work on these sheets. Answer completely, but be concise. *A normal probability table is attached.*

Part 1: Multiple Choice. *Circle the letter corresponding to the best answer.*

1. You want to compute a 96% confidence interval for a population mean. Assume that the population standard deviation is known to be 10 and the sample size is 50. The value of z* to be used in this calculation is
   (a) 1.960
   (b) 1.645
   (c) 1.7507
   (d) 2.0537
   (e) None of the above. The answer is ________________.

2. You want to estimate the mean SAT score for a population of students with a 90% confidence interval. Assume that the population standard deviation is $\sigma = 100$. If you want the margin of error to be approximately 10, you will need a sample size of
   (a) 16
   (b) 271
   (c) 38
   (d) 1476
   (e) None of the above. The answer is ________________.

   $1.645 \frac{100}{\sqrt{n}} \leq 10$

3. A significance test gives a *P*-value of 0.04. From this we can
   (a) Reject H_0 at the 1% significance level
   (b) Reject H_0 at the 5% significance level
   (c) Say that the probability that H_0 is false is 0.04
   (d) Say that the probability that H_0 is true is 0.04
   (e) None of the above. The answer is ________________.

4. A significance test was performed to test the null hypothesis H_0: $\mu = 2$ versus the alternative H_a: $\mu \neq 2$. The test statistic is $z = 1.40$. The *P*-value for this test is approximately .
   (a) 0.16
   (b) 0.08
   (c) 0.003
   (d) 0.92
   (e) 0.70
   (f) None of the above. The answer is ________________.

   Table: $1 - 0.9192 = 0.0808$

   ☆ two-sided ☆

5. You have measured the systolic blood pressure of a random sample of 25 employees of a company located near you. A 95% confidence interval for the mean systolic blood pressure for the employees of this company is (122, 138). Which of the following statements gives a valid interpretation of this interval?

(a) Ninety-five percent of the sample of employees has a systolic blood pressure between 122 and 138.
(b) Ninety-five percent of the population of employees has a systolic blood pressure between 122 and 138.
(c) If the procedure were repeated many times, 95% of the resulting confidence intervals would contain the population mean systolic blood pressure.
(d) The probability that the population mean blood pressure is between 122 and 138 is .95.
(e) If the procedure were repeated many times, 95% of the sample means would be between 122 and 138.
(f) None of the above. The answer is __________________.

6. An analyst, using a random sample of $n = 500$ families, obtained a 90% confidence interval for mean monthly family income for a large population: ($600, $800). If the analyst had used a 99% confidence coefficient instead, the confidence interval would be:

(a) Narrower and would involve a larger risk of being incorrect
(b) Wider and would involve a smaller risk of being incorrect
(c) Narrower and would involve a smaller risk of being incorrect
(d) Wider and would involve a larger risk of being incorrect
(e) Wider but it cannot be determined whether the risk of being incorrect would be larger or smaller

7. To determine the reliability of experts used in interpreting the results of polygraph examinations in criminal investigations, 280 cases were studied. The results were:

		True Status: Innocent	True Status: Guilty
Examiner's Decision	"Innocent"	131	15
	"Guilty"	9	125

If the hypotheses were H_0: suspect is innocent vs. H_a: suspect is guilty then we could estimate the probability of making a Type II error as:

(a) 15/280
(b) 9/280
(c) 15/140
(d) 9/140
(e) 15/146

Fail to reject H_0,
Ha is true

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1. Patients with chronic kidney failure may be treated with dialysis which may cause retention of phosphorous. A study of the nutrition of dialysis patients measured the level of phosphorous in the blood of several patients on six occasions. Here are the data for one patient. $\bar{X} = 5.3667$

5.6	5.1	4.6	4.8	5.7	6.4

These measurements can be considered an SRS. Actual phosphorus level varies normally with standard deviation .9.

a. Give a 90% confidence interval for the mean blood phosphorus level.

$$\bar{X} \pm z^* \frac{\sigma}{\sqrt{n}} = 5.3667 \pm 1.645 \frac{0.9}{\sqrt{6}} \longrightarrow (4.7623, 5.9711)$$

b. What is the margin of error?

$$\pm 1.645 \frac{0.9}{\sqrt{6}} = \pm 0.6044$$

c. What sample size would you have to use to make the margin of error a maximum of .4?

$$1.645 \frac{0.9}{\sqrt{n}} \leq .4 \rightarrow n \approx 14$$

2. Consider the hypotheses: $H_o: \mu = 4.8$ and $H_a: \mu \neq 4.8$

a. Does the above sample give evidence against the null hypothesis at the 10% significance level?

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{5.3667 - 4.8}{0.9/\sqrt{6}} = 1.5424$$

P-value $= 0.0615$
$2(0.0615) = 0.123$
$0.123 > 0.10$, we fait to reject H_0.

b. Does the sample give strong evidence against the null hypothesis?

No; since $0.123 > 0.10$, we do not have enough evidence to reject H_0. Therefore, we cannot say the level is not 4.8.

c. Suppose in another sample the mean was 5.7. Does this sample give strong evidence against the null hypothesis? YES!

$$\frac{5.7 - 4.8}{0.9/\sqrt{6}} = 2.4495$$

P-value $= 0.00715$ | $2(0.00715) = 0.0143$
$0.0143 < 0.05$

d. Is the outcome 5.7 significant at the 5% level? At the 1% level?

YES! NO!

e. Suppose another sample outcome gives a test statistic of -2.3. Would the sample outcome be significant at the 5% level? YES!

$z = -2.3 \rightarrow$ P-value $= 0.0107$ | $2(0.0107) = 0.0214 < 0.05$

f. What is the probability of a Type I error if this test is done at the 1% significance level?

$\alpha = 0.01$

~~g.~~ What is the probability of a Type II error if the actual $\mu_x = 5.36$?

$$-2.576 \leq \frac{\bar{X} - 4.8}{0.9/\sqrt{6}} \leq 2.576$$

$3.8535 \leq \bar{X} \leq 5.7465$ | $\beta = 0.009994$

~~h.~~ What is the power of a 1% significance test against the alternative 5.36?

$1 - \beta = 0.9900$