

Show all work/and or explain your answers completely! Answer in context (a sentence or the probability statement with the random variable defined in words!)

1. A problem in a test given to small children asks them to match each of three pictures of animals to the word identifying that animal. If a child assigns the three words at random to the three pictures, find the probability distribution for X , the number of correct matches.

X	0	1	2	3
$P(X)$	$1/8$	$3/8$	$3/8$	$1/8$

2. Five balls, numbered 1 through 5, are placed in an urn. Two balls are randomly drawn from the urn, and the numbers noted. Find the probability distribution for ...

- (a) X the largest number drawn

X	1	2	3	4	5
$P(X)$	$0/10$	$1/10$	$2/10$	$3/10$	$4/10$

- (b) Y the sum of the numbers
(1,2) (1,3) (1,4) (1,5)
(2,3) (2,4) (2,5) (3,4)
(3,5) (4,5)

Y	3	4	5	6	7	8	9
$P(Y)$	$1/10$	$1/10$	$2/10$	$2/10$	$2/10$	$1/10$	$1/10$

A box contains three \$1 bills, two \$5 bills, one \$10 and one \$20. A bill is drawn at random. Construct a probability distribution for the amount of money drawn. What is the mean and the standard deviation of this distribution?

X	1	5	10	20
$P(X)$	$3/7$	$2/7$	$1/7$	$1/7$

$$\mu_x = 6.14$$

$$\sigma_x = 6.4238$$

4. A new game at the State Fair allows a player to pull a bill out of a box. The box contains ten \$1's, five \$2's, three \$5's, one \$10 and one \$100. The game costs \$20 to play. What is the expected result of the game? Is the game fair?

X	-19	-18	-15	-10	80
$P(X)$	$10/20$	$5/20$	$3/20$	$1/20$	$1/20$

$$E(X) = -\$12.75$$

5. Ethel and Frank's game involves two rolled dice. If the sum is 2 or 12, the player wins \$20. If the sum is 7, the player wins \$5. The game costs \$3 to play. What is the expected result of the game?

X	17	2	-3
$P(X)$	$2/36$	$6/36$	$28/36$

$$E(X) = -\$1.06$$

6. A shoe store manager computes the probabilities of selling shoes for each size. Find the mean and variance of the distribution.

Size (Y)	8	9	10	11	12	13
$P(Y)$	0.13	0.39	0.23	0.14	0.08	0.03

$$\mu_x = 9.74 \quad \sigma_x^2 = 1.5724 \quad \sigma_x = 1.2540$$

7. Choose an American household at random and let the random variable X be the number of persons living in the household. If we ignore the few households with more than 7 inhabitants, the probability distribution of X is as shown below.

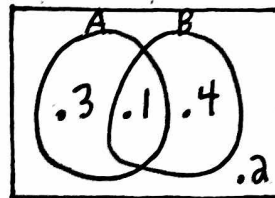
Inhabitants	1	2	3	4	5	6	7
Probability	0.25	0.32	0.17	0.15	0.07	0.03	0.01

(a) $P(X \geq 5) = .11 \Rightarrow 11\%$
 (b) $P(X > 5) = .04 \Rightarrow 4\%$

(c) $P(2 < X \leq 4) = .32 \Rightarrow 32\%$
 (d) $P(X \neq 1) = .75 \Rightarrow 75\%$

8. When testing private wells for two types of impurities, it was found that 20% had neither type, 40% had A, and 50% had B (some had both). Let Y represent the number of impurities found in a randomly selected well. Find the probability distribution for Y .

Y	0	1	2
$P(Y)$.2	.7	.1



$P(A \cup B) = .8$

9. You are playing a board game in which the severity of a penalty is determined by rolling three dice and adding the spots on the up-faces. The dice are all balanced so that each face is equally likely, and three dice fall independently.

(a) Give the sample space for the sum X of spots $S = \{3, 4, \dots, 17, 18\}$

X	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$P(X)$	$1/216$	$3/216$	$6/216$	$10/216$	$15/216$	$21/216$	$25/216$	$27/216$	$27/216$	$25/216$	$21/216$	$15/216$	$10/216$	$6/216$	$3/216$	$1/216$

(b) Find $P(X=5)$ $6/216 = 1/36 = .0278 \Rightarrow 2.78\%$

- (c) If $X_1, X_2,$ and X_3 are the number of spots on the up-faces of the three dice, then $X = X_1 + X_2 + X_3$. Use this fact to find the mean μ_X and the standard deviation σ_X without finding the distribution of X . (Start with the distribution of each of the X_i)

$\mu_{X_1} = 3.5$ | $\mu_X = 3.5 + 3.5 + 3.5$ | $\sigma_{X_1}^2 = 2.9166$ | $\sigma_X^2 = 8.751$
 | $\mu_X = 10.5$ | $\sigma_{X_1} = 1.7078$ | $\sigma_X = 2.958$

10. One (six-sided) fair die is rolled, and one (two-sided) fair coin is tossed. If the coin turns up heads, then the number of spots showing on the die is the value (score) for that trial. If the coin turns up tails, then twice the number of spots showing on the die is the value (score) for that trial. Determine the outcomes in the sample space, and construct a probability distribution table. Find the expected value of the following experiment. If I were to invite you to play a game where I would pay you your score in dollars, what should I charge you to play? Using your ticket price, how many games would have to be played in order for me to have an expected profit of \$100?

$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

X	1	2	3	4	5	6	8	10	12
$P(X)$	$1/12$	$2/12$	$1/12$	$2/12$	$1/12$	$2/12$	$1/12$	$1/12$	$1/12$

$E(X) = \$5.25$

Charge \$5.50 (profit \$0.25 per game); Play 400 games to gain \$100.

Directions: Work on these sheets.

Part 1: Multiple Choice. Circle the letter corresponding to the best answer.

A psychologist studied the number of puzzles subjects were able to solve in a five-minute period while listening to soothing music. Let X be the number of puzzles completed successfully by a subject. X had the following distribution:

X	1	2	3	4
Probability	0.2	0.4	0.3	0.1

- Using the above data, what is the probability that a randomly chosen subject completes at least 3 puzzles in the five-minute period while listening to soothing music?
(a) 0.3
(b) 0.4
(c) 0.6
(d) 0.9
(e) The answer cannot be computed from the information given.
- Using the above data, $P(X < 3)$ is
(a) 0.3
(b) 0.4
(c) 0.6
(d) 0.9
(e) The answer cannot be computed from the information given.
- Using the above data, the mean μ of X is
(a) 2.0
(b) 2.3
(c) 2.5
(d) 3.0
(e) The answer cannot be computed from the information given.
- Which of the following random variables should be considered continuous?
(a) The time it takes for a randomly chosen woman to run 100 meters
(b) The number of brothers a randomly chosen person has
(c) The number of cars owned by a randomly chosen adult male
(d) The number of orders received by a mail order company in a randomly chosen week
(e) None of the above

5. Let the random variable X represent the profit made on a randomly selected day by a certain store. Assume that X is normal with mean \$360 and standard deviation \$50. What is the value of $P(X > \$400)$?
- (a) 0.2119
 (b) 0.2881
 (c) 0.7881
 (d) 0.8450
 (e) The answer cannot be computed from the information given.

$$Z = 0.8 \Rightarrow 1 - 0.7881$$

Part 2: Free Response

Answer completely, but be concise. Write sequentially and show all steps.

The probability that 0, 1, 2, 3, or 4 people will seek treatment for the flu during any given hour at an emergency room is shown in the distribution.

X	0	1	2	3	4
$P(X)$	0.12	0.25	0.32	0.24	0.06

6. What does the random variable count or measure?

$X =$ the number of people who seek treatment

7. What is the mean of X ? for the flu during any given hour.

$$\underline{\mu_X = 1.85}$$

8. What is the variance and standard deviation of X ?

$$\sigma_X^2 = 1.193275$$

$$\underline{\sigma_X = 1.0924}$$

9. If a player rolls two dice and gets a sum of 2 or 12, he wins \$20. If the person gets a 7, he wins \$5. The cost to play the game is \$3. Find the expectation of the game.

$X =$ Amount of money made

X	17	2	-3
$P(X)$	2/36	6/36	28/36

$$\underline{E(X) = -\$1.06}$$