

Ch. 8 Review

AP Statistics

Name: Mr. Morton

When a computerized generator is used to generate random digits, the probability that any particular digit in the set $\{0, 1, 2, \dots, 9\}$ is generated on any individual trial is $1/10 = 0.10$. Suppose that we are generating random digits one at a time and are interested in tracking the occurrences of the digit 0.

1. The random variable X is geometric. Define X . $X = \frac{\# \text{ of random digits until a } 0 \text{ occurs.}}{}$
2. Verify that this describes a geometric setting.

B - Binomial ✓

I - Independent ✓

R - Repeat Trials ✓

S - Probability success is the same ✓

3. Find the probability that the first 0 occurs as the fifth random digit generated.

$$P(X=5) = (.9)^4(.1) = \underline{0.06561}$$

4. Construct a probability distribution table for X (up through $X = 5$).

| | | | | | |
|--------|----|-----|------|-------|-------|
| X | 1 | 2 | 3 | 4 | 5 |
| $P(X)$ | .1 | .09 | .081 | .0729 | .0656 |

5. How many digits would you expect to have to generate in order to observe a 0?

$$\mu_x = \frac{1}{p} = \frac{1}{.1} = \underline{10 \text{ generated digits}}$$

Directions: Work on these sheets. A random digit table is attached.

Part 1: Multiple Choice. Circle the letter corresponding to the best answer.

1. In a large population of college students, 20% of the students have experienced feelings of math anxiety. If you take a random sample of 10 students from this population, the probability that exactly 2 students have experienced math anxiety is

(a) 0.3020

(b) 0.2634

(c) 0.2013

(d) 0.5

(e) 1

(f) None of the above

$$P(X=2)$$

2. Refer to the previous problem. The standard deviation of the number of students in the sample who have experienced math anxiety is

(a) 0.0160

(b) 1.265

(c) 0.2530

(d) 1

(e) .2070

$$\sigma_x = \sqrt{np(1-p)}$$

3. In a certain large population, 40% of households have a total annual income of \$70,000. A simple random sample of 4 of these households is selected. What is the probability that 2 or more of the households in the survey have an annual income of over \$70,000?

(a) 0.3456

(b) 0.4000

(c) 0.5000

(d) 0.5248

(e) The answer cannot be computed from the information given.

$$P(X \geq 2)$$

4. A factory makes silicon chips for use in computers. It is known that about 90% of the chips meet specifications. Every hour a sample of 18 chips is selected at random for testing. Assume a binomial distribution is valid. Suppose we collect a large number of these samples of 18 chips and determine the number meeting specifications in each sample. What is the approximate mean of the number of chips meeting specifications?

(a) 16.20

(b) 1.62

(c) 4.02

(d) 16.00

(e) The answer cannot be computed from the information given.

$$\mu_x = np$$

5. Which of the following are true statements?
- The expected value of a geometric random variable is determined by the formula $(1-p)^{n-1}p$.
 - If X is a geometric random variable and the probability of success is .85, then the probability distribution of X will be skewed left, since .85 is closer to 1 than to 0.
 - An important difference between binomial and geometric random variables is that there is a fixed number of trials in a binomial setting, and the number of trials varies in a geometric setting.
- (a) I only
 (b) II only
 (c) III only
 (d) I, II, and III
 (e) None of the above gives the complete set of true responses.

Part 2: Free Response

Answer completely, but be concise. Write sequentially and show all steps.

According to government data, 20% of employed women have never been married.

6. What is the random variable X of interest here? Define X . Is X normal, binomial, or geometric?

$X = \#$ of employed women who have never been married.

7. If 10 employed women are selected at random, what is the probability that 2 or fewer have never been married? $n=10, p=.2$

$P(X \leq 2) = 0.6778$

8. What are the mean and standard deviation of X ?

| | |
|-------------------------------------|--|
| $\mu_X = np = 10(.20)$ | $\sigma_X = \sqrt{np(1-p)}$ $\sigma_X = 1.2649$ women |
| <u>$\mu_X = 2$ women</u> | |

9. Find the probability that the number of employed women who have never been married is within 1 standard deviation of its mean.

$2 \pm 1.2649 \Rightarrow (0.7351, 3.2649)$

$P(1 \leq X \leq 3) = 0.7718$

10. Describe the four conditions that describe a binomial setting.

B - binomial ✓

I - independent ✓

N - fixed number of trials ✓

S - probability success is the same ✓

A quarterback completes 44% of his passes.

11. Explain how you could use a table of random numbers to simulate this quarterback attempting 20 passes.

Assign 00-43 as completion, 44-99 as incomplete.
No skips, repeats okay, read table in double digits, stop after 20 labels are selected.

12. Explain how you could use a TI-83 to simulate this quarterback attempting 20 passes.

SEED: 123 → rand

Randint(00, 99, 20) ✓

13. Using your scheme from either (11) or (12), simulate 20 passes. If you use the random digit table, begin on line 149. If you use the TI-83, first enter 149 → rand to seed your random number generator, and indicate which one you use. List the numbers generated and circle the "successes." Calculate the percent of passes completed.

8 52 57 91 31 65 92 66 95 43
13 39 52 33 46 36 32 91 5 35

$$\text{passes completed} = \frac{10}{20} = .5$$

50%

14. What is the probability that the quarterback throws 3 incomplete passes before he has a completion?

$$P(X=4) = (.56)^3(.44) = \underline{0.07727}$$

15. How many passes can the quarterback expect to throw before he completes a pass?

$$\mu_x = \frac{1}{p} = \frac{1}{.44} = \underline{2.2727 \text{ passes}}$$

16. Use *two* methods to determine the probability that it takes more than 5 attempts before he completes a pass.

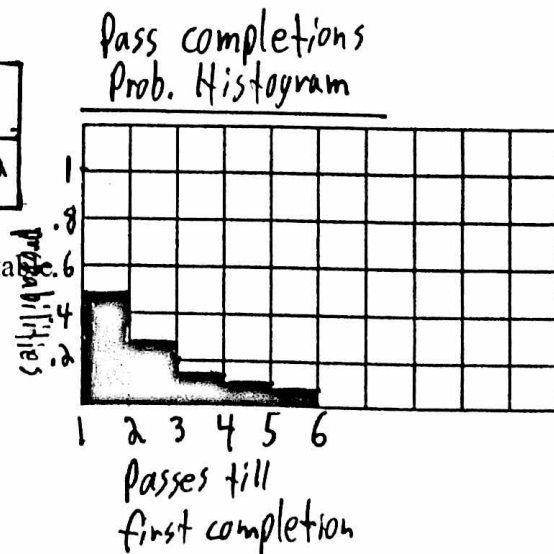
$$P(X > 5) = (1 - .44)^5 = \underline{0.05507} \star$$

$$P(X > 5) = 1 - P(X \leq 5) = \underline{0.05507} \star$$

17. Construct a probability distribution table (out to $n = 6$) for the number of passes attempted before the quarterback has a completion.

| | | | | | | |
|------|-----|-------|--------|--------|--------|-------|
| X | 1 | 2 | 3 | 4 | 5 | 6 |
| P(X) | .44 | .2464 | .13798 | .07727 | .04327 | .0242 |

18. Sketch a probability histogram (out to $n = 6$) for the table you constructed in the previous problem.



Directions: Work on these sheets. A random digit table is attached.

Part 1: Multiple Choice. Circle the letter corresponding to the best answer.

1. A dealer in the Sands Casino in Las Vegas selects 40 cards from a standard deck of 52 cards. Let Y be the number of red cards (hearts or diamonds) in the 40 cards selected. Which of the following best describes this setting:
- (a) Y has a binomial distribution with $n = 40$ observations and probability of success $p = 0.5$.
- (b) Y has a binomial distribution with $n=40$ observations and probability of success $p = 0.5$, provided the deck is shuffled well.
- (c) Y has a binomial distribution with $n=40$ observations and probability of success $p = 0.5$, provided after selecting a card it is replaced in the deck and the deck is shuffled well before the next card is selected.
- (d) Y has a normal distribution with mean $p = 0.5$.
2. In a certain large population, 40% of households have a total annual income of over \$70,000. A simple random sample is taken of 4 of these households. Let X be the number of households in the sample with an annual income of over \$70,000 and assume that the binomial assumptions are reasonable. What is the mean of X ?
- (a) 1.6
- (b) 28,000
- (c) 0.96
- (d) 2, since the mean must be an integer
- (e) The answer cannot be computed from the information given.
- $\mu_x = np$
3. The probability that a three-year-old battery still works is 0.8. A cassette recorder requires four working batteries to operate. The state of batteries can be regarded as independent, and four three-year-old batteries are selected for the cassette recorder. What is the probability that the cassette recorder operates?
- (a) 0.9984
- (b) 0.8000
- (c) 0.5904
- (d) 0.4096
- (e) The answer cannot be computed from the information given.
- $P(X=4)$
4. Twenty percent of all trucks undergoing a certain inspection will fail the inspection. Assume that trucks are independently undergoing this inspection, one at a time. The expected number of trucks inspected before a truck fails inspection is
- (a) 2
- (b) 4
- (c) 5
- (d) 20
- (e) The answer cannot be computed from the information given.
- $\mu_x = \frac{1}{p}$